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# Global Optimization in Practice: An Application to Interactive Multiple Objective Linear Programming

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**Abstract.** A multiple objective linear programming problem (P) involves the simultaneous maximization of two or more conflicting linear objective functions over a nonempty polyhedron $X$. Many of the most popular methods for solving this type of problem, including many well-known interactive methods, involve searching the efficient set $X_E$ of the problem. Generally, however, $X_E$ is a complicated, nonconvex set. As a result, concepts and methods from global optimization may be useful in searching $X_E$. In this paper, we will explain in theory, and show via an actual application to citrus rootstock selection in Florida, how the potential usefulness of the well-known interactive method STEM for solving problem (P) in this way, can depend crucially upon how accurately certain global optimization problems involving minimizations over $X_E$ are solved. In particular, we will show both in theory and in practice that the choice of whether to use the popular but unreliable "payoff table" approach or to use one of the lesser known, more accurate global optimization methods to solve these problems can determine whether STEM succeeds or fails as a decision aid. Several lessons and conclusions of transferable value derived from this research are also given.

**Key words:** Multiple objective linear programming, Optimization over the efficient set, Interactive methods, Global optimization, Citrus rootstock selection.

## 1. Introduction

The multiple objective linear programming problem (P) involves the simultaneous maximization of $p \geq 2$ conflicting linear objective functions over a nonempty polyhedron $X$. Typically, this problem is solved by a decision maker (DM) who, with the aid of an analyst, searches the feasible region $X$ of the problem for a *most preferred* solution.

The concept of an efficient solution has played an important role in the analysis and solution of this problem. An efficient solution $x^\circ$ for problem (P) is an element of $X$ such that no other element of $X$ exists which achieves a value at least as large as $x^\circ$ in every objective function and a strictly larger value than $x^\circ$ in at least one objective function. It can be shown that except under very unusual circumstances, there will always exist a most preferred solution to the problem that is also an efficient solution.

Consequently, many of the most popular approaches for analyzing and solving a multiple objective linear programming problem search all, or at least some, of the efficient solution set $X_E$ in order to find a most preferred solution. Included among these types of approaches are, for instance, the vector maximization approach, interactive approaches, and several others (see, for instance, books and general surveys by Cohon [29], Evans [35], Goicoechea et al. [41], Ringuest [55], Rosenthal [56], Sawaragi et al. [58], Steuer [63], Yu [70, 71], Zeleny [73] and references therein). In the past 25 years, a vast number of applications of multiple objective linear programming has been reported in a wide variety of disciplines, including academic planning [39], macroeconomic analysis [5], nutrition planning [2, 19], public policy making [8], capital budgeting [26], meat processing [63], finance and banking [33], forest management [64], production planning [40], logistics [50] and many others too numerous to mention. In view of these observations, it has become especially important to develop practical methods for searching the efficient set of a multiple objective linear program.

Unfortunately, searching the efficient set $X_E$ of a multiple objective linear program for a most preferred solution is generally a very challenging task. While a variety of reasons for this difficulty exist, chief among them is that $X_E$ is generally a large, complicated nonconvex subset of the feasible region $X$. This suggests that to generate or search $X_E$ or portions of $X_E$, concepts and methods from global optimization may be useful.

Yet, with only a few exceptions, the concepts and methods developed in the past 25 years for multiple objective linear programming have relied mainly on using local search ideas from traditional linear and convex programming. Since these local search ideas were never intended to provide a search of a large, complicated nonconvex set such as $X_E$, none of the methods developed to date for solving multiple objective linear programming problems has proven to be wholly adequate (see Benson and Sayin [22], Shin and Ravindran [60], Steuer [63] and references therein for further details).

In recent years, a resurgence of interest in the *interactive* strategies for searching $X_E$ has become apparent [1, 56, 60, 63]. An interactive method for searching $X_E$ for a most-preferred solution consists of a finite number of DM-machine interactions that generate a discrete sample of points from $X_E$. During a typical iteration, a computer program first finds a point in $X_E$ by solving an appropriate single-objective optimization problem. Next, the DM is asked to assess his or her relative preference for this point. Based upon this information, the single-objective optimization problem is modified, and another iteration begins. These DM-machine interactions are repeated until the DM indicates that the current solution is a most-preferred solution to the problem. A large number of interactive algorithms using a variety of approaches for problem (P) have been proposed. For details concerning these algorithms, the reader may consult any of a number of survey papers [1, 35, 56, 60] and textbooks [28, 41, 58, 63, 70].

In this paper, we will show in both theory and practice that the use of appropriate global optimization techniques can be crucial to the achievement of success in applied multiple objective linear programming. In particular, we will explain in theory and show via an actual application how the potential usefulness of the popular STEM [7] interactive method for solving a multiple objective linear program can depend critically on how accurately certain global optimization problems involving minimizations over $X_E$ are solved. The application that we will use to illustrate this point is one that we carried out ourselves for the purposes of developing procedures for solving the citrus rootstock selection problem in Florida [18]. Lessons and conclusions of transferable value that can be derived from this research will also be explained.

The organization of this paper is as follows. In Section 2, notation and preliminaries concerning problem (P), the need for global optimization over $X_E$ in solving problem (P), and the STEM algorithm will be given. In Section 3, we will explain why, in theory, researchers have been concerned with the accuracy with which the difficult global minimizations over $X_E$ called for by STEM are solved. We will also point out that in practice, however, these concerns have not yet been addressed. In Section 4 we briefly outline the citrus rootstock selection problem in Florida and the multiple objective linear programming model (ROOT) that we have constructed for it. Section 5 describes two interactive solution attempts of the model (ROOT) that we undertook using STEM. In the first attempt, which failed, a simple but unpredictable 'payoff table' procedure [7] was used to attempt to estimate the minima over $X_E$ required by STEM. In the second attempt, which succeeded, a much more accurate global optimization heuristic recently developed by Benson and Sayin [20] was used to provide these estimates. Lessons and conclusions of transferable value are given in the last section.

## 2. Notation and preliminaries

Adopting the notation of Geoffrion [38], we may represent a multiple objective linear programming problem (P) by

$$\text{VMAX: } Cx, \quad \text{subject to } x \in X,$$

where $C$ is a $p \times n$ matrix whose rows $c_i, i = 1, 2, \ldots, p$, are the coefficients of the $p$ linear objective functions of the problem, and $X \subseteq \mathbb{R}^n$ is a nonempty polyhedron. Since the objective functions are, in general, conflicting, the 'maximization' aspect of problem (P) is not *a priori* well defined. Usually, a DM, together with an analyst, seeks a *most preferred* solution $x^*$, if one exists, to problem (P), i.e. a solution $x^*$ that belongs to $X$ and maximizes $v[\langle c_1, x\rangle, \langle c_2, x\rangle, \ldots, \langle c_p, x\rangle]$ over $X$, where $v : \mathbb{R}^p \to \mathbb{R}$ is the value function of the DM for problem (P).

Unfortunately, the value function $v$ of the DM is usually not explicitly available or computable (cf. Sawaragi et al. [58], Shin and Ravindran [60] and Steuer [63]). In cases such as these, vector maximization methods or interactive algorithms for problem (P) are usually used. The theoretical premise behind these algorithms is that, in the absence of $v$, to find a most preferred solution, the DM can search appropriate subsets of $X$ rather than the entire set $X$. Two of these subsets are defined as follows.

DEFINITION 2.1. The set $X_E$ of all *efficient* (or *nondominated*) solutions $x^\circ$ for problem (P) is the set of all points $x^\circ \in X$ for which there exists no $x \in X$ such that $Cx \geq Cx^\circ$ and $Cx \neq Cx^\circ$.

DEFINITION 2.2. The set $\overline{X}_E$ of all *weakly-efficient* (or *weakly-nondominated*) solutions $\overline{x}^\circ$ for problem (P) is the set of all points $\overline{x}^\circ \in X$ for which there exists no $x \in X$ such that $Cx > C\overline{x}^\circ$.

Notice from Definitions 2.1 and 2.2 that $X_E \subset \overline{X}_E$. The rationale for searching $X_E$ or $\overline{X}_E$ for a most preferred solution comes, in part, from the following result.

PROPOSITION 2.1. *If $X$ is bounded and the value function $v : \mathbb{R}^P \to \mathbb{R}$ of the DM for problem (P) is coordinatewise nondecreasing, then there exists a most preferred solution of the DM for problem (P) which belongs to $X_E$ and, hence, to $\overline{X}_E$.*

Proposition 2.1 is a simple result to prove. For a proof of this proposition and of related results, see Benson and Aksoy [16], Henig [42], Soland [62] and Steuer [63].

Although both $X_E$ and $\overline{X}_E$ are connected sets and are typically smaller than $X$, both are, unfortunately, generally quite complicated, nonconvex sets [10, 23, 70, 72]. In particular, both $X_E$ and $\overline{X}_E$ consist of unions of faces of $X$, that, in general, may have great variations in their dimensions and locations within $X$ (cf., e.g., [3, 15, 34, 63, 70]). Consequently, searching $X_E$ or $\overline{X}_E$ for a most preferred solution is, in fact, a difficult global optimization problem. In spite of this, it seems that very few vector maximization methods or interactive methods for searching $X_E$ or $\overline{X}_E$ take explicit account of this important aspect of the problem (cf. Armann [4] and Benson and Sayin [22]). Instead, most of these methods are based upon local search ideas that can be implemented by simplex-type or combinatorial search techniques [3, 34, 44, 63, 70, 72].

There are however, a few methods for searching $X_E$ or $\overline{X}_E$ that take the global nature of the search into account, either explicitly or implicitly. Some of these methods are vector maximization methods [4, 22]. Others use the interactive approach (cf., e.g., [6, 7, 47, 52]).

The interactive methods that acknowledge the global nature of the search frequently do so implicitly by calling for the computation of the so-called range of compromise for each objective function. This range is defined as follows.

DEFINITION 2.3. For each $i = 1, 2, \ldots, p$, the *range of compromise* for objective function $i$ of problem (P) is a subinterval of the extended real number line given by $[m_i, M_i] = \{t \in \mathbb{R} | m_i \leq t \leq M_i\}$, where $m_i$ and $M_i$ denote the infimum and the supremum, respectively, of $\langle c_i, x\rangle$ over $X_E$.

Notice that when $X$ is bounded, the range of compromise for each objective function of problem (P) is also bounded.

Those interactive algorithms that use the concept of the range of compromise do so for a variety of reasons. In some cases, the range of compromise helps the DM to set goals or aspiration levels called for by the algorithm. In other cases, this range simply helps the DM to answer the preference-related questions called for by the algorithm. Other possible uses involve ranking or eliminating objective functions (see Weistroffer [66], Dessouky et al. [32], Isermann and Steuer [45] and Benson [11] for details.)

Notice that regardless of the purpose for which an algorithm calls for computing the ranges of compromise, the need for these ranges implicitly acknowledges the global nature of searching $X_E$. This becomes apparent by noting that computing each range of compromise necessitates solving two global optimization problems over $X_E$. The first of these is the maximization of an objective function $\langle c_i, x\rangle$ over $X_E$. It is well known that this can be easily accomplished via linear programming, i.e., without special global optimization tools (cf., e.g., Benayoun et al. [7], Evans and Steuer [36]). However, the second problem, calculating the minimum value of an objective function $\langle c_i, x\rangle$ over $X_E$, involves the global minimization of a linear function over a nonconvex set. This is a difficult global optimization problem which, as in most other classes of global optimization problems, is typically plagued by having numerous local optima that are not global [14, 17, 43, 48]. Further confounding the situation is the fact that the feasible region $X_E$ of this problem cannot be expressed in the traditional mathematical programming format as a system of functional inequalities.

One of the earliest and most popular interactive algorithms for problem (P), called the STEP Method (or simply STEM) [7], calls for computing the ranges of compromise for each objective function. Therefore, to implement STEM, several global minimization problems over $X_E$ must be solved.

As a final preliminary to presenting our main research results, we will briefly explain the steps and properties of the STEM method for problem (P). Amplifications and further details concerning STEM can be found in [7, 18, 63].

The idea behind STEM is to interactively search with the DM for a most preferred solution to problem (P) by generating and examining points in $\overline{X}_E \supseteq X_E$. During each STEM iteration $h$, a weakly-efficient point $x^h$ is first generated by solving a certain weighted-minimax problem. Next, the DM examines this point to see if it is a most preferred solution. If so, the algorithm stops. Otherwise, the DM specifies an index set $K \subset \{1, 2, \ldots, p\}$ of criteria whose current values $\langle c_k, x^h\rangle, k \in K$, he is willing to allow to decrease in order to potentially attain increases in one or more of the other criteria values $\langle c_k, x^h\rangle, k \in \{1, 2, \ldots, p\}\backslash K$. In addition, for each $k \in K$, the DM specifies the maximum amount $\Delta_k > 0$ that he is willing to allow criterion $k$ to decrease. Using $K$ and the values $\Delta_k, k \in K$, constraints of the weighted-minimax problem are appropriately modified in preparation for generating a new weakly-efficient point in the next iteration.

To use STEM, $X$ must be bounded. Therefore, we assume henceforth that $X$ is bounded. Recall that in this case, each range of compromise $[m_k, M_k], k = 1, 2, \ldots, p$, is finite. In addition, in this case, the vectors $M^T = [M_1, M_2, \ldots, M_p]$ and $m^T = [m_1, m_2, \ldots, m_p]$ are usually called the *ideal point* and the *nadir point*, respectively, for problem (P). The steps of the STEM algorithm may be stated as follows.

STEM ALGORITHM FOR PROBLEM (P)

*Initialization Step 0.*

*Step 0.1.* Compute the ideal point $M$ and the nadir point $m$ for problem (P).

*Step 0.2.* For each $k = 1, 2, \ldots, p$ compute the value of $L_k = \sum_{j=1}^{n} |c_{kj}|$.

*Step 0.3.* For each $k = 1, 2, \ldots, p$, compute the criterion weight $\pi_k$ according to the formula

$$\pi_k = [(M_k - m_k)/\max(|M_k|, |m_k|)](L_k/\|c_k\|), \tag{1}$$

where $\|\cdot\|$ denotes the Euclidean norm.

*Step 0.4.* Set $K = \phi, h = 1$ and $X^1 = X$, and go to Iteration $h$.

*Iteration $h$, $h \geq 1$.*

*Step h.1.* Find an optimal solution $(x^h, t^h) \in \mathbb{R}^{n+1}$ to the linear program (PK) given by

$$\begin{aligned} \min\ & t, \\ \text{s.t. } t \ \geq\ & \left(\pi_k / \sum_{j=1}^{p} \pi_j\right)(M_k - \langle c_k, x\rangle), \quad k \in \{1, 2, \ldots, p\}\backslash K, \\ x\ \in\ & X^h, \\ t\ \geq\ & 0. \end{aligned}$$

*Step h.2.* Compute $\langle c_k, x^h\rangle, k = 1, 2, \ldots, p$, and, given $x^h$ and these objective function values, ask the DM if he feels satisfied that the algorithm can stop with $x^h$ as the final solution. If so, stop with $x^h$ as a most preferred solution. If not, continue.

*Step h.3.* Ask the DM to identify the index set $K \subset \{1, 2, \ldots, p\}$ of the criteria that he is willing to relax in order to potentially gain increases in one or more of the other criteria. For each $k \in K$, ask the DM to specify the maximum amount $\Delta_k > 0$ that he is willing to relax criterion $k$.

*Step h.4.* Set $X^{h+1} = \{x \in X | \langle c_k, x\rangle \geq \langle c_k, x^h\rangle - \Delta_k,\ k \in K; \langle c_k, x\rangle \geq \langle c_k, x^h\rangle,\ k \in \{1, 2, \ldots, p\}\backslash K\}$ and set $h = h + 1$. Go to Iteration $h$.

At each iteration $h$ of STEM, the linear program (PK) finds a feasible solution $x^h$ to problem (P) which minimizes the maximum of the weighted deviations $w_k(M_k - \langle c_k, x\rangle)$ of the values of the criteria $k \in \{1, 2, \ldots, p\}\backslash K$ from their associated ideal point values $M_k$, $k \in \{1, 2, \ldots, p\}\backslash K$, subject to $x \in X^h$, where for each $k$, the weight $w_k$ is given by

$$w_k = \left(\pi_k / \sum_{j=1}^{p} \pi_j\right). \tag{2}$$

It is easy to see from (1)–(2) that for each $k \in \{1, 2, \ldots, p\}$, if $M_k > m_k$, then $w_k$ will be positive. This, in turn, can be shown to imply that whenever $M > m$, each solution $x^h$ generated by STEM belongs to $\overline{X}_E$. Notice that due to the constraints of problem (PK), $x^h$ is not generally an extreme point of $X$.

For each $k \in \{1, 2, \ldots, p\}$, the purpose of the term $(L_k/\|c_k\|)$ used in formula (1) for $\pi_k$ in the algorithm is to rescale the coefficients of $c_k$ in the linear program (PK). The intention is to yield coefficient vectors of similar magnitudes [7, 63]. The other term in this formula defines the relative range of compromise for criterion $k \in \{1, 2, \ldots, p\}$. Thus, the intention of (1)–(2) is to yield weights $w_k, k \in \{1, 2, \ldots, p\}$, for use in problem (PK) that have larger values for those objective functions with larger relative ranges of compromise over $X_E$, rather than for those with large-magnitude coefficients. Such weights are expected to increase the ability of the DM to effectively use STEM [63]. Notice from (1)–(2) that the accuracy of the assessment of each weight $w_k$ depends, in part, upon how accurately the corresponding criterion $\langle c_k, x\rangle$ is minimized over $X_E$ in computing the nadir point $m$ in Step 0.1.

During each iteration $h$ of STEM, the relaxation quantities $\Delta_k, k \in K$, chosen by the DM in Step h.3 are based upon his priorities and aspiration levels. Typically, to help guide the DM in his choices for the values of $\Delta_k, k \in K$, the analyst is encouraged to suggest to the DM that he also refer to the ranges of compromise $[m_k, M_k], k \in K$ [58].

The STEM algorithm has received relatively-more attention and study than most other interactive algorithms for problem (P). Although several reasons can be cited for this, chief among them is that STEM is relatively easy for the DM to use and understand, that it does not confine its search of $\overline{X}_E$ (and thus of $X_E$) to extreme points of $X$, and that it has proven useful in certain practical instances [58, 63, 65].

## 3. Global optimization in STEM: theoretical issues

Notice that except for finding the ideal point $M$ and the nadir point $m$ in Step 0.1, the computational steps in the STEM algorithm can be easily implemented via simple matrix-vector operations and linear programming methods. However, as we have seen, the global maximizations and minimizations over $X_E$ called for in calculating $M$ and $m$, respectively, are not standard problems.

Finding $M$ requires finding, for each $k \in \{1, 2, \ldots, p\}$, the optimal value $M_k$ of the global optimization problem

$$\max\langle c_k, x\rangle, \quad \text{subject to } x \in X_E.$$

From the time that STEM was proposed, it has been known that for each $k \in \{1, 2, \ldots, p\}$, $M_k$ is alternatively given by

$$M_k = \max\langle c_k, x\rangle, \quad \text{subject to } x \in X, \tag{3}$$

i.e., that $M_k$ can be found by standard linear programming methods [7, 63].

However, to find the nadir point $m$, one must calculate, for each $k = 1, 2, \ldots, p$, the optimal value $m_k$ of the difficult global minimization problem $(P_k)$ over $X_E$ given by

$$\min\langle c_k, x\rangle, \quad \text{subject to } x \in X_E.$$

Researchers have been suspecting for several years that, at least in theory, unless the nadir point $m$ is calculated with sufficient accuracy, various algorithms that rely on $m$ to solve problem (P) should not be expected to necessarily find a most preferred solution to the problem. The STEM method has been consistently included among the algorithms that could fail for this reason (cf., e.g., Ghiassi et al. [40], Weistroffer [66], Isermann and Steuer [45], and Reeves and Reid [54]).

In the case of the STEM method, incorrect estimates of $m$ have a direct and an indirect effect. The direct effect, as seen from (1)–(2), is that the minimax weights $w_k$, $k = 1, 2, \ldots, p$, used in problem (PK) are improperly calculated. The indirect effect is that the DM, armed with inaccurate ranges of compromise, is more likely to internally develop inappropriate aspiration levels for the objective functions. This, in turn, is likely to cause the DM difficulty in choosing appropriate relaxation quantities $\Delta_k$ for the objective functions $k \in K$ in Step h.4 [58]. Taken together, these phenomena reduce the likelihood that STEM will generate solutions for problem (P) that are attractive to the DM.

The theoretical response to the concern over the availability of methods for accurately determining the nadir point $m$ has been quite strong, especially in recent years. In particular, a variety of exact global optimization algorithms (cf., e.g., Benson [11–13], Benson and Sayin [21], Bolintineanu [24], Dauer and Fosnaugh [31], Fulop [37], Philip [53] and references therein) and heuristic methods (see, e.g., Benayoun et al. [7], Benson and Sayin [20], Dauer [30], Dessouky et al. [32], Korhonen et al. [48] and references therein) capable of determining or estimating $m$ has been proposed.

In spite of the theoretical concern that nadir points should be globally estimated with accuracy sufficient for their use in algorithms for problem (P), it appears that, in actual practice, users of STEM have not addressed this concern. An examination of the literature concerning the use of STEM reveals that practitioners continue to rely on an unpredictable rule of thumb called the ‘payoff table method’ [7] to estimate nadir point values [32, 45, 54, 63, 66].

In the payoff table method, for each $k = 1, 2, \ldots, p$, the value $\hat{m}_k$ given by

$$\hat{m}_k = \min\{\langle c_k, x^i\rangle | i = 1, 2, \ldots, p\} \tag{4}$$

is used to estimate the optimal value of problem ($P_k$), where, for each $i \in \{1, 2, \ldots, p\}$, $x^i$ maximizes $\langle c^i, x\rangle$ over $X$. The vector $\hat{m}^T = [\hat{m}_1, \hat{m}_2, \ldots, \hat{m}_p]$ is thus the estimate provided by this method for the actual nadir point $m$. It is well known, however, that the entries in $\hat{m}$ are not reliable. They can either underestimate or overestimate the corresponding values in $m$, sometimes quite significantly [32, 45, 48, 66].

Although unreliable, the payoff table method is well known by practitioners of multiple criteria decision making. It is also easy for anyone with access to linear programming software to use, especially in contrast to many of the complex optimization algorithms that have been proposed for finding exact, global optima for problem ($P_k$). Furthermore, only theoretical, not actual, warnings have been published to the effect that, in conjunction with STEM, the payoff table method can sabotage the effort to successfully solve problem (P). For these reasons, in actual practice, practitioners of STEM have resisted addressing the well-founded concern over properly estimating $m$ when using STEM in applied decision making.

## 4. The citrus rootstock application

Our experience applying STEM to a multiple objective citrus rootstock selection model (ROOT) that we constructed for Florida will show how in practice, as well as in theory, the usefulness of STEM can depend crucially upon how accurately the global optimization problems ($P_k$), $k = 1, 2, \ldots, p$, used to find the nadir point are solved. To explain these applications of STEM, a basic understanding of the citrus rootstock selection model (ROOT) is needed. In this section, a very brief overview of Florida’s citrus rootstock selection problem and the model (ROOT) will be given. Readers interested in a complete presentation of this problem and the model (ROOT) should consult [18].

Prior to the 1970s and 1980s, citrus rootstock selection in Florida was not a critical issue, since essentially only two different kinds of rootstocks, Rough lemon and Sour orange, were used. In fact, these two rootstocks had been used with great success for many years [46, 49].

However, during the 1970s and 1980s, the situation changed drastically. There were three reasons for this change. First, a new citrus disease of unknown origin, called *citrus blight*, suddenly appeared in Florida and devastated large numbers of trees. Second, marked increases in a viral disease called *tristeza* occurred, which weakened and killed many trees. Third, Florida experienced unprecedented increases in the frequency and severity of fruit-damaging freezes in its citrus groves during the 1980s [27].

As a result of these three factors, major shifts in rootstock selection and increased research into alternate rootstocks and their characteristics began during the 1980s [25, 57, 67–69]. Rootstocks such as Cleopatra mandarin, Swingle citrumelo and Carrizo citrange were introduced, studied, and planted in Florida for the first time. Some rootstocks were introduced that could resist diseases such as *Phytophthora* [61] and *tristeza*, but could not achieve the high yields or fruit quality of other rootstocks. Other rootstocks that were introduced had high yields but could not resist factors such as blight or cold.

While a variety of citrus rootstocks is now used in Florida, there is no single rootstock that is superior in all characteristics. As a result, Florida citrus growers now plant mixed portfolios of different types of rootstocks in their groves based upon the priorities that they have for achieving various goals.

Generally, the overall goals of a citrus grower in Florida today are to plant a grove that will have a high yield of fruit, excellent fruit quality, and tolerance of freeze, blight and tristeza [27]. Growers routinely also show ongoing concern that groves adequately resist phytophthora and drought [27], since both are still prevalent in Florida.

To reflect these goals and concerns, we have constructed a multiple objective linear programming model (ROOT) for solving the citrus rootstock selection problem for a typical Florida grove. In this model, the grower has a choice of $n$ different types of citrus rootstocks and, for each one, $m$ different types of scions (buds) to graft onto each rootstock. Each scion-rootstock combination creates a citrus tree with distinct characteristics. Thus, in the general model (ROOT), a Florida citrus grower can choose to plant up to $mn$ different types of trees in his grove. Let $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ denote indices representing the $m$ types of scions and $n$ types of rootstocks, respectively, available to the grower. For each $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ we will call a tree created by budding a scion $i$ onto a rootstock $j$ a tree of 'type $i - j$'.

There are $mn$ decision variables in the model (ROOT). These variables are denoted $x_{ij}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, where, for each $i$ and $j$,

$$x_{ij} = \text{no. of trees of type } i - j \text{ to plant.}$$

The model is concerned with a period of $S$ years following the planting, where $S$ is a parameter usually of size 10 or more. There are four objective functions to be maximized in model (ROOT). These objective functions, denoted $z_1$, $-z_2$, $-z_3$, and $-z_4$, are linear functions of the decision variables, where

$z_1 =$ the expected annual grove yield in pounds-solids during the period of $S$ years,

$z_2 =$ the weighted-average cold damage level of the trees in the grove due to freezes,

$z_3$ = the no. of trees affected by blight during the period of $S$ years, and

$z_4$ = the no. of trees visibly-affected by tristeza during the period of $S$ years.

Here, ‘pounds-solids’ is a standard unit of yield used in the citrus industry that depends upon both the quantity and the quality of the fruit produced [18]. To measure the cold damage levels of individual trees needed to construct the function $z_2$, we used an interval scale of 1–5 proposed by Rouse et al. [57]. In this scale, greater damage from cold corresponds to larger scale values (see Benson et al. [18] for details).

The model (ROOT) has linear constraints. One of these constraints defines the total number of trees to be planted in the grove in question. The others specify upper bounds on the aggregate degrees of susceptibility of the grove to various diseases (including Phytophthora) and to drought that will be allowed. In the model, the number of trees to plant and the upper bounds on degrees of susceptibility allowed are parameters whose values are specified by the DM prior to the solution phase.

## 5. Global optimization in STEM: the practical need

To test the usefulness of the citrus rootstock selection model (ROOT), we twice attempted to use STEM to solve an application of (ROOT) to a typical grove of 10,000 citrus trees in the Fort Pierce area of Florida. In the first attempt, following the lead of other researchers (cf., e.g., [7, 41, 51, 63, 73]), we used the rule of thumb given by the payoff table method to estimate the global optimal values of the problems ($P_k$), $k = 1, 2, 3, 4$, needed for finding the nadir point $m$. To our dismay, this solution attempt failed. However, suspecting that the sources for this failure were poor estimates for $m_k$, $k = 1, 2, 3, 4$, provided by the payoff table method, we undertook a second solution attempt. In this attempt, which succeeded, we used a much more accurate global optimization method by Benson and Sayin [20] to estimate the optimal solutions and values to the problems ($P_k$), $k = 1, 2, 3, 4$. These results show that in practice, as well as in theory, the usefulness of the STEM interactive method for solving problem (P) can depend crucially upon how accurately the global minimizations over $X_E$ called for by STEM are solved.

In both STEM solution attempts, one of the authors (McClure) served as the DM for the STEM algorithm. Part of his job as Executive Vice President of Becker Grove Inc. (Fort Pierce, Florida) is to directly manage all grove operations, including the selection of citrus rootstocks. Another author (Lee), a specialist in multiple criteria decision making methods, acted as the analyst in charge of guiding the DM through the steps of STEM and executing the mathematics and computer routines called for by STEM. The Fort Pierce citrus data required for this application of the model (ROOT) was collected from various research studies and historical records [18]. The time horizon used in the application was $S = 12$ years.

Prior to applying STEM, the DM explained that in the Fort Pierce area, high and equal priorities are usually placed on maximizing the number of pounds-solids yielded and on minimizing tristeza damage. This is because pounds-solids yielded directly correlates with income, and, in the Fort Pierce area, forecasts call for tristeza to become an even more serious problem in the next ten years than it is now.

On the other hand, in the Fort Pierce area, only moderate priority is generally placed on minimizing damage due to blight, and even less priority is given to minimizing cold damage. This is because blight, although always a possibility, is less of a threat in the Fort Pierce area than in other areas of Florida, and freezes in this area are extremely rare.

Following are summaries of the two solution attempts, with insights into how and why more accurate global optimization to find the nadir point was the difference between success and failure. For further details regarding these two solution attempts, please consult [18].

### 5.1. Failed solution attempt with payoff table method

In the first attempt to solve the Fort Pierce application, we used linear programming and (3) to find the ideal point $M$, and we used the payoff table method to estimate the nadir point $m$. This resulted in the values $M^T = [95107, -2.770, -415.6, 0.0]$ for the ideal point and in the estimate $\hat{m}$ for the nadir point, where

$$\begin{aligned} \hat{m}_1 &= 89750, \\ \hat{m}_2 &= -3.589, \\ \hat{m}_3 &= -993.8, \text{ and} \\ \hat{m}_4 &= -1375.0. \end{aligned}$$

For each $k = 1, 2, 3, 4$, using the payoff table estimate $\hat{m}_k$ for $m_k$, formulas (1)–(2) were used as called for by the STEM algorithm to calculate the minimax weights $w_k$, $k = 1, 2, 3, 4$, needed to define the linear programs (PK) solved in STEM. This resulted in weight values of $w_1 = 0.0475$, $w_2 = 0.1916$, $w_3 = 0.3814$ and $w_4 = 0.3795$.

With these values for $\hat{m}_k$ and $w_k$, $k = 1, 2, 3, 4$, the analyst and the DM executed the STEM algorithm for the Fort Pierce application. The analyst solved the necessary linear programming problems on an IBM personal computer via the simplex method implementation in the LINDO [59] software package. The DM terminated the STEM procedure after six iterations. Together, the analyst and the DM spent approximately 75 minutes executing these six iterations of STEM.

Table I summarizes the criterion value results for each of these six STEM iterations. In particular, for each iteration $h$, this table gives the index set $K$ of the relaxed criteria, the criterion values $\langle c_k, x^h \rangle$, $k = 1, 2, 3, 4$, of the weakly-efficient point found by STEM, and, in square brackets, the lower limits $\langle c_k, x^h \rangle - \Delta_k$, $k \in K$, to which the DM was willing to relax the criteria $k \in K$.

Contrary to our hopes, the DM explained that he terminated the STEM process in this case after six iterations out of a sense of frustration rather than because he

Table I. Criterion values and relaxation limits for STEM using payoff tables

| Iteration | $K$ | Criterion values and relaxation limits $k=$ 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $h=1$ | {2, 3} | 76128 | −2.86 | −450.0 | −12.8 |
| | | | [−3.40] | [−600.0] | |
| 2 | {2, 3} | 81685 | −3.18 | −600.0 | −7.5 |
| | | | [−3.40] | [−700.0] | |
| 3 | {2, 4} | 84034 | −3.30 | −700.0 | −6.2 |
| | | | [−3.40] | | [−100.0] |
| 4 | {2, 3} | 84034 | −3.30 | −658.1 | −100.0 |
| | | | [−3.40] | [−800.0] | |
| 5 | {2} | 86159 | −3.40 | −800.0 | −4.97 |
| | | | [−3.50] | | |
| 6 | — | 86159 | −3.41 | −790.5 | −4.97 |

felt he had found a most preferred solution. After iteration six, the DM explained, he felt from viewing the results in Table 1 that he could not reasonably expect to find a most preferred solution or a nearly most preferred solution via STEM. He was especially disappointed that he could not find solutions with larger values in criterion number 1 (expected annual yields).

To help discover why this STEM solution attempt failed, we first asked the DM to explain the rationale for his responses during the STEM iterations. From his explanations, two general themes emerged.

First, the primary thrust of the DMs responses was to attempt to find a feasible solution with an expected annual grove yield of 87500 pounds-solids or more. The DM felt that this was an essential element of any solution, based upon his knowledge of grove yields in the Fort Pierce area. Furthermore, he felt that the model could reasonably be expected to yield many such solutions, since, according to the payoff table estimate, the yields for the efficient solutions, for instance, range from $\hat{m}_1 = 89750$ to $M_1 = 95107$ pounds-solids.

Second, to attempt to find solutions with larger expected yields, the DM concentrated on relaxing his aspiration levels for cold damage and blight, but never to levels below $-3.50$ and $-800.0$, respectively. He concentrated on lowering these two aspiration levels rather than his tristeza aspiration level because of the higher priority that he placed on resistance to tristeza than on resistance to cold or blight, and because in most iterations he was less concerned with the cold damage and blight criterion values than with the tristeza criterion value. His decision never to relax the aspiration levels for cold damage and for blight below $-3.50$ and $-800.0$, respectively, arose from his judgement that, based upon the payoff table ranges $[\hat{m}_2, M_2] = [-3.589, -2.770]$ and $[\hat{m}_3, M_3] = [-993.8, -415.6]$ for these two criteria, values below $-3.50$ and $-800.0$ would be unacceptably close to their smallest possible values over the efficient set.

After interviewing the DM and obtaining this information, we decided to attempt to assess the accuracy of the estimates $\hat{m}_k$, $k = 1, 2, 3$. We knew how, in theory, inaccurate global estimates of the components of $m$ could inhibit the successful performance of STEM (cf. Section 3). We focused on the accuracy of $\hat{m}_k$, $k = 1, 2, 3$, because, from the comments of the DM, we felt that he considered criteria 1-3 more heavily than the fourth criterion during the STEM iterations.

We first found that $\hat{m}_1 = 89750 > m_1$, i.e. that $\hat{m}_1$ is an overestimate of the true minimum $m_1$ of $z_1$ over $X_E$. To deduce this, we first tested each weakly-efficient solution $x^h$, $h = 1, 2, \ldots, 6$, generated by STEM for efficiency by using a simple linear programming test [9]. We found each solution to be efficient. Next, we observed from Table I that each of the six criterion values $z_1$ in the column $k = 1$ satisfied $z_1 < \hat{m}_1$. Since each solution $x^h$ is efficient, this immediately led us to the conclusion that $\hat{m}_1 > m_1$.

Notice that since $\hat{m}_1 > m_1$, the use of $\hat{m}_1$ in (1) instead of $m_1$ led to a value for $w_1$ of 0.0475 in (2) which underestimates the true value of $w_1$. Also, from the constraints of the linear program (PK) solved in Step h.1 of STEM, it can be seen that using inappropriately-small values for $w_1$ there will generally yield solutions to problem (ROOT) with smaller values for $z_1$ than would be obtained by using the correct value for $w_1$. Therefore, the fact that $\hat{m}_1 = 89750$ overestimates $m_1$ could have, among other things, significantly contributed to the inability of the DM to find solutions with expected annual yields at or above his aspiration level of 87500 pounds-solids.

Next, we found that for $k = 2, 3, \hat{m}_k \geq m_k$, i.e., that $\hat{m}_k$ either overestimates or correctly estimates $m_k$. To deduce this, for each $k = 2, 3$, we tested the solution $x^{i_k} \in X$ satisfying $\hat{m}_k = \langle c_k, x^{i_k} \rangle$ generated by formula (4) of the payoff table method for efficiency by again using the test in [9]. For each $k = 2, 3$, we found that $x^{i_k} \in X_E$. For each $k = 2, 3$, since $\hat{m}_k = \langle c_k, x^{i_k} \rangle$, this led us to conclude that $\hat{m}_k \geq m_k$. However, unlike the case for $k = 1$, we could not definitely show mathematically that $\hat{m}_k$ strictly overestimates the true minimum $m_k$ for $k = 2, 3$.

Although $\hat{m}_2 = m_2$ and $\hat{m}_3 = m_3$ were possible, the unsatisfactory STEM results and the comments of the DM caused us to suspect that, in fact, either $\hat{m}_2 > m_2$ or $\hat{m}_3 > m_3$, or both, were true. To see why, notice that by using (3) and the payoff table formula (4) for calculating $M_1$ and $\hat{m}_1$, respectively, we had found actual feasible solutions with expected yields of $M_1 = 95107$ pounds-solids and of $\hat{m}_1 = 89750$ pounds-solids. This proved that rootstock planting choices exist which more than satisfy the DM's aspiration level for $z_1$ of 87500 pounds-solids. Yet, during the STEM procedure, in spite of the DM's continual reductions of his aspiration levels for the cold damage and blight criteria, STEM failed to find any plans with expected yields of more than 86159 pounds-solids (cf. Table I). Recall, however, that based upon the payoff table ranges of compromise $[\hat{m}_k, M_k], k = 2, 3$, for these two criteria, the DM had decided never to reduce these two aspiration levels below $-3.50$ and $-800.0$, respectively. Given smaller values for one or both of $\hat{m}_k$, $k = 2, 3$, it is possible that the DM would have been willing to allow one or both of these aspiration levels to be reduced further, in which case STEM may have succeeded in generating solutions with adequate expected yields. Taken together, these observations suggest the possibility that the values for one or both of $\hat{m}_k$, $k = 2, 3$, are inappropriately large.

### 5.2. Successful solution attempt with improved global optimization

Since the attempt to solve the Fort Pierce application by using STEM with the standard payoff table method failed to yield a satisfactory solution, we made a second attempt to solve the problem. As before, we used STEM as the main tool. However, having discovered that payoff table overestimates of $m_k$, $k = 1, 2, 3$, could have significantly contributed to the failure of the first solution attempt, we used a more sophisticated global optimization procedure by Benson and Sayin to estimate these values for this attempt. This procedure, a face search heuristic for global optimization over $X_E$ [20], was chosen for its computational ease, proven accuracy, and amenability to user control.

From the Benson–Sayin method, we found the estimate $\hat{\hat{m}}$ for $m$, where

$$\begin{aligned} \hat{\hat{m}}_1 &= 76000 < \hat{m}_1, \\ \hat{\hat{m}}_2 &= -3.717 < \hat{m}_2, \\ \hat{\hat{m}}_3 &= -1380.9 < \hat{m}_3, \text{ and} \\ \hat{\hat{m}}_4 &= -1375.0 = \hat{m}_4. \end{aligned}$$

Since $\hat{\hat{m}} \geq m$ is guaranteed [20], these results show that for each $k = 1, 2, 3$, the Benson–Sayin estimate $\hat{\hat{m}}_k$ is superior to the payoff table estimate $\hat{m}_k$. Similarly, since $\hat{\hat{m}} \geq m$, these results confirmed our suspicion that the payoff table estimates $\hat{m}_2$ and $\hat{m}_3$ strictly overestimate $m_2$ and $m_3$, respectively, and they reconfirmed our discovery that $\hat{m}_1 > m_1$.

For each $k = 1, 2, 3, 4$, using the Benson–Sayin estimate for $\hat{\hat{m}}_k$ for $m_k$, we found via (1)–(2) STEM weight values of $w_1 = 0.1389, w_2 = 0.1752, w_3 = 0.3752$ and $w_4 = 0.3106$. For each $k = 1, 2, 3, 4$, using the new estimate of $\hat{\hat{m}}_k$ of $m_k$ and the new weight value $w_k$, the analyst and the DM once again executed the STEM algorithm for the Fort Pierce application. After two iterations, which together took approximately twenty minutes, the DM terminated the STEM procedure.

Table II. Criterion values and relaxation limits for STEM using the Benson–Sayin algorithm

| | | Criterion Values and Relaxation Limits | | | |
|---|---|---|---|---|---|
| Iteration | $K$ | $k =$ 1 | 2 | 3 | 4 |
| $h = 1$ | {2,3} | 79233 | −3.05 [−3.60] | −485.3 [−900.0] | −31.5 |
| 2 | — | 88464 | −3.51 | −900.0 | −13.2 |

Table II summarizes the criterion value results for each of the two iterations for this solution attempt. The format of the table is the same as that of Table I.

Unlike in the first solution attempt, the DM terminated the second STEM solution attempt with a final solution $x^f$ and criteria values with which he was quite pleased. The DM explained that he felt sufficiently confident that this solution was either a most-preferred solution or close enough to a most-preferred solution to terminate the process.

By quizzing the DM on the rationale for his responses during the second solution attempt, and by comparing the results of this solution attempt with those of the first attempt, we concluded that the second attempt succeeded where the first did not because of the improved accuracy of the Benson–Sayin global estimates of $m_k$, $k = 1, 2, 3$, compared to the payoff-table estimates. This improvement manifested itself in several ways. Two of these were especially crucial.

First, by using $\hat{\hat{m}}_1 = 76000$ as an estimate for $m_1$ in (1), instead of $\hat{m}_1 = 89750$ as in the payoff table approach, the weight $w_1$ computed via (2) with the Benson–Sayin approach had a significantly-larger value (0.1389) than the value (0.0475) that it had with the payoff table approach. This helped to give the DM the ability to find solutions via the Benson–Sayin approach with higher yields than those found via the payoff table approach. Mathematically, this ability can be explained by the fact that as larger values for $w_1$ (see (2)) are used in the linear program (PK) solved in Step h.1 of STEM, solutions $(x^h, t^h)$ with larger values for $z_1$ will, in general, be generated.

Second, by using the Benson–Sayin values $\hat{\hat{m}}_2 = -3.717$ and $\hat{\hat{m}}_3 = -1380.9$ as estimates for $m_2$ and $m_3$, instead of using the larger estimates $\hat{m}_2 = -3.589$ and $\hat{m}_3 = -993.8$ found via the payoff table approach, the DM obtained a more accurate assessment of the ranges of $z_k$, $k = 2, 3$, over $X_E$. Based upon this knowledge, he explained to us that in the second solution attempt, he was willing to relax his aspiration levels for the cold damage and blight criteria further than in the first attempt. In particular, in the second solution attempt he was willing to relax the cold damage and blight aspiration levels to values as low as $-3.60$ and $-900.0$, respectively, rather than only to $-3.50$ and $-800.0$ as before. Indeed, the DM implemented these relaxations in the first iteration of the second solution attempt (see Table II). Together with the larger value of $w_1$, these deeper relaxations allowed STEM to generate the high expected-yield solution in iteration 2 that could not be found in the first solution attempt.

After the execution of the second STEM solution procedure, the DM was quite convinced that the STEM solution process using the Benson–Sayin global optimization heuristic was superior to both the first solution process, which used the payoff table method, and to the current informal rootstock selection process used at Becker Groves. He cited several reasons for this, including the ease of the process and the ability it gives the DM to actively explore and learn about the available solutions and their tradeoffs with respect to criteria values [18]

The DM also was quite convinced that the final solution $x^f$ given by the second solution attempt is superior to the planning schemes generally used at Becker Grove. In particular, while the yields, cold damage level, and tristeza level expected using $x^f$ (see Table II, row $h = 2$) are essentially the same as recent plantings at Becker Grove, the blight protection expected using $x^f$ is significantly greater (see [18] for further details).

## 6. Main conclusions

There are a number of conclusions and lessons of transferable value that can be derived from this research. The major ones are as follows.

1. In both theory and practice, the use of appropriate global optimization methods can be crucial to the achievement of success in applied multiple objective decision making.
2. In actual applications, inaccurate global optimization estimates of nadir values when using the STEM method for interactive multiple objective linear programming can and have frustrated decision makers. The use of inappropriate global optimization methods in these instances can and has led to unsatisfactory results.
3. Exact and approximate global optimization procedures are available which, when used to find nadir points in STEM, considerably increase the probability of allowing a decision maker to successfully find a most preferred solution to an applied multiple objective linear programming model via the STEM method.
4. In general, the use of the unreliable payoff table method to globally estimate nadir points in real-world multiple objective linear programming applications is to be discouraged. Instead, the use of any of a number of more accurate and reliable global optimization methods is recommended.

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